Closing Wed: HW_3D,4A,4B(6.2/3,6.4)

## 6.4 Work *(continued)*

*Recall from last lecture:*

If a task involves a constant force moving a given distance: Work = Force·Distance

If work or distance change during the task, then label, break into subdivisions (find a pattern) and compute

$$\text{Work} = \lim_{n\to\infty} \sum_{i=1}^{n} (\text{Force} \ \cdot \ \text{Distance})$$

$$= \int_a^b (Force)(Distance)$$

**PROBLEM TYPE 1** ("Leaky Bucket"):
An single object is lifted, but gets lighter (or heavier) as it is lifted.

FORCE = $f(x_i)$, DIST = $\Delta x$

**PROBLEM TYPE 2** ("Stack of Books")
A bunch of objects are stacked up and each gets lifted a different distance.
FORCE=(density)(horiz. length or volume)
DIST = depends on labels
(typically $x$ or $a - y$)

*Examples*:

1. (Chains/Cables) You are lifting a heavy chain to the top of a building. The chain has a density of 3 lbs/foot. The chain hangs over the side by 25 feet before you start pulling it up. How much work is done in pulling the chain all the way to the top?

Example:

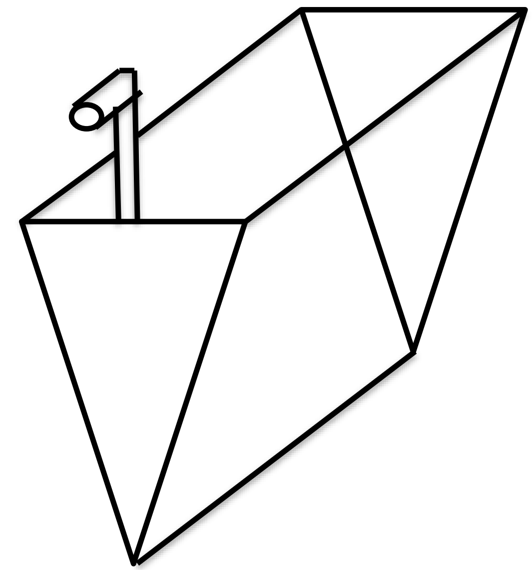
A 50 foot cable with density 4 lbs/ft is hanging over the side of a tall building.

Find the total work done in lifting the cable half way up.

2. (Pumping Liquid) You are pumping water out of a tank. The tank is a rectangular box with a base of 2 ft by 3 ft and height of 10ft.
The density of water is 62.5 lbs/ft$^3$.

If the tank starts full, how much work is done in pumping all the water to the top and out over the side?

*Example*:
Consider the tank show at right.
The height is 3 meters, the width at the top is 2 meters and the length is 6 meters.  Also we are pumping the water up to 4 meters above the ground (1 meter above the top edge).
If it starts full, how much work is done to pump it all out?



## 6.5 Average Value

The average value of the *n* numbers:

$$y_1, y_2, y_3, \ldots, y_n$$

is given by

$$\frac{y_1+y_2+y_3+\cdots+y_n}{n} = y_1\frac{1}{n} + \cdots + y_n\frac{1}{n}.$$

Goal: We want the average value of **all** the y-values of some function y = f(x) over an interval x = a to x = b.

*Derivation*:

1. Break into *n* equal subdivisions

$$\Delta x = \frac{b-a}{n}, \text{ which means } \frac{\Delta x}{b-a} = \frac{1}{n}$$

2. Compute *y*-value at each tick mark

$$y_1 = f(x_1), y_2 = f(x_2), \ldots, y_n = f(x_n)$$

3. Ave $\approx f(x_1)\frac{\Delta x}{b-a} + \cdots + f(x_n)\frac{\Delta x}{b-a}$

$$\text{Average} \approx \frac{1}{b-a}\sum_{i=1}^{n} f(x_i)\Delta x$$

Thus, we can define

$$\text{Average} = \frac{1}{b-a}\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

Which means the exact average y-value of y = f(x) over x = a to x = b is

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$$